Anderson • Sweeney • Williams • Camm • Cochran • Fry • Ohlmann

An Introduction to Management Science:

Quantitative Approaches to Decision Making 14e

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An Introduction to Management Science: Quantitative Approaches to Decision Making, Fourteenth Edition
David R. Anderson, Dennis J. Sweeney, Thomas A. Williams, Jeffrey D. Camm, James J. Cochran, Michael J. Fry, Jeffrey W. Ohlmann

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Cover Designer: Beckmeyer Design

Cover Image: iStockphoto.com/alienforce

Intellectual Property
  Analyst: Christina Ciaramella
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WCN: 02-200-203

Library of Congress Control Number: 2014947009

ISBN: 978-1-111-82361-0

Cengage Learning
20 Channel Center Street
Boston, MA 02210
USA

Printed in the United States of America
Print Number: 01 Print Year: 2014

Dedication

To My Parents
Ray and Ilene Anderson
DRA

To My Parents
James and Gladys Sweeney
DJS

To My Parents
Phil and Ann Williams
TAW

To My Parents
Randall and Jeannine Camm
JDC

To My Wife
Teresa
JJC

To My Parents
Mike and Cynthia Fry
MJF

To My Parents
Willis and Phyllis Ohlmann
JWO

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Preface

We are very excited to publish the fourteenth edition of a text that has been a leader in the field for nearly 25 years. The purpose of this fourteenth edition, as with previous editions, is to provide undergraduate and graduate students with a sound conceptual understanding of the role that management science plays in the decision-making process. The text describes many of the applications where management science is used successfully. Former users of this text have told us that the applications we describe have led them to find new ways to use management science in their organizations.

An Introduction to Management Science: Quantiative Approaches to Decision Making, 14e is applications oriented and continues to use the problem-scenario approach that is a hallmark of every edition of the text. Using the problem scenario approach, we describe a problem in conjunction with the management science model being introduced. The model is then solved to generate a solution and recommendation to management. We have found that this approach helps to motivate the student by demonstrating not only how the procedure works, but also how it contributes to the decision-making process.

From the first edition we have been committed to the challenge of writing a textbook that would help make the mathematical and technical concepts of management science understandable and useful to students of business and economics. Judging from the responses from our teaching colleagues and thousands of students, we have successfully met the challenge. Indeed, it is the helpful comments and suggestions of many loyal users that have been a major reason why the text is so successful.

Throughout the text we have utilized generally accepted notation for the topic being covered so those students who pursue study beyond the level of this text should be comfortable reading more advanced material. To assist in further study, a references and bibliography section is included at the back of the book.

CHANGES IN THE FOURTEENTH EDITION

We are very excited about the changes in the fourteenth edition of Management Science and want to explain them and why they were made. Many changes have been made throughout the text in response to suggestions from instructors and students. While we cannot list all these changes, we highlight the more significant revisions.

New Members of the ASW Team

Prior to getting into the content changes, we want to announce that we have some changes in the ASW author team for *Management Science*. Previous author Kipp Martin decided to pursue other interests and will no longer be involved with this text. We thank Kipp for his previous contributions to this text. We have brought on board three new outstanding authors who we believe will be strong contributors and bring a thoughtful and fresh view as we move forward. The new authors are James Cochran, University of Alabama, Michael Fry of the University of Cincinnati, and Jeffrey Ohlmann, University of Iowa. You may read more about each of these authors in the brief bios which follow.

Updated Chapter 9: Project Scheduling

Within this chapter, the section on considering variability's impact on project completion time has been significantly revised. The new discussion maintains the emphasis on the critical path in estimating the probability of completing a project by a specified deadline, but generalizes this calculation to also consider the other paths through the project network. Also, Appendix 9.1 has been added to show how to find a cumulative probability for a normally distributed random variable; the normal distribution is commonly used to describe the completion time for sequences of activities.

Updated Chapter 6: Distribution and Network Models

This chapter has been updated and rearranged to reflect the increased importance of supply chain applications for quantitative methods in business. Transportation and transshipment models are grouped into a single section on supply chain models. This chapter better represents the current importance of supply chain models for business managers. All models in this chapter are presented as linear programs. In keeping with the theme of this book, we do not burden the student with solution algorithms in the chapter. Details on many of the solution algorithms used in this text can still be found in the Web chapters for this text.

Updated Chapter 13: Decision Analysis

This chapter has been updated with a new section on Utility Theory to complement the previous material on decision analysis.

Updated Chapter 15: Time Series Analysis and Forecasting

We have updated our discussion of trends and seasonality in Chapter 15. We now focus on the use of regression to estimate linear trends and seasonal effects. We have also added a discussion on using the Excel LINEST function to estimate linear trends and seasonal effects in Appendix 15.1 at the end of this chapter. These revisions better represent industry approaches to these important topics.

Management Science in Action

The Management Science in Action vignettes describe how the material covered in a chapter is used in practice. Some of these are provided by practitioners. Others are based on articles from publications such as *Interfaces* and *OR/MS Today*. We updated the text with over 20 new Management Science in Action vignettes in this edition.

Cases and Problems

The quality of the problems and case problems is an important feature of the text. In this edition we have added over 45 new problems and 3 new case problems.

COMPUTER SOFTWARE INTEGRATION

To make it easy for new users of LINGO or Excel Solver, we provide both LINGO and Excel files with the model formulation for every optimization problem that appears in the body of the text. The model files are well-documented and should make it easy for the user to understand the model formulation. Microsoft Excel 2010 and 2013 both use an updated version of Excel Solver that allows all problems in this book to be solved using the standard version of Excel Solver. LINGO 14.0 is the version used in the text.

In an Appendix 12.2 at the end of Chapter 12, we have replaced Crystal Ball™ with Analytic Solver Platform to construct and solve simulation models. In Appendix 13.1 at the end of Chapter 13, we have replaced the TreePlan software with Analytic Solver Platform to create decision trees.

FEATURES AND PEDAGOGY

We have continued many of the features that appeared in previous editions. Some of the important ones are noted here.

Annotations

Annotations that highlight key points and provide additional insights for the student are a continuing feature of this edition. These annotations, which appear in the margins, are designed to provide emphasis and enhance understanding of the terms and concepts being presented in the text.

Notes and Comments

At the end of many sections, we provide Notes and Comments designed to give the student additional insights about the methodology and its application. Notes and Comments include warnings about or limitations of the methodology, recommendations for application, brief descriptions of additional technical considerations, and other matters.

Self-Test Exercises

Certain exercises are identified as self-test exercises. Completely worked-out solutions for those exercises are provided in an appendix at the end of the text. Students can attempt the self-test exercises and immediately check the solution to evaluate their understanding of the concepts presented in the chapter.

ANCILLARY TEACHING AND LEARNING MATERIALS

For Students

Print and online resources are available to help the student work more efficiently.

- **LINGO.** A link to download an educational version of the LINGO software is available on the student website at www.cengagebrain.com.
- **Analytic Solver Platform.** Instructions to download an educational version of Frontline Systems' (the makers of Excel Solver) Analytic Solver Platform are included with the purchase of this textbook. These instructions can be found within the inside front cover of the text.

For Instructors

Instructor support materials are available to adopters from the Cengage Learning customer service line at 800-423-0563 or through www.cengage.com. Instructor resources are available on the Instructor Companion Site, which can be found and accessed at login.cengage.com, including:

- **Solutions Manual.** The Solutions Manual, prepared by the authors, includes solutions for all problems in the text.
- **Solutions to Case Problems.** These are also prepared by the authors and contain solutions to all case problems presented in the text.

- **PowerPoint Presentation Slides.** Prepared by John Loucks of St. Edwards University, the presentation slides contain a teaching outline that incorporates graphics to help instructors create more stimulating lectures.
- **Test Bank.** Cengage Learning Testing Powered by Cognero is a flexible, online system that allows you to:
  - author, edit, and manage test bank content from multiple Cengage Learning solutions,
  - create multiple test versions in an instant,
  - deliver tests from your LMS, your classroom or wherever you want. The Test Bank is also available in Microsoft Word.

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For more information about instructor resources, please contact your Cengage Learning Consultant.

ACKNOWLEDGMENTS

We owe a debt to many of our colleagues and friends whose names appear below for their helpful comments and suggestions during the development of this and previous editions. Our associates from organizations who supplied several of the Management Science in Action vignettes make a major contribution to the text. These individuals are cited in a credit line associated with each vignette.

Art Adelberg
CUNY Queens College

Joseph Bailey
University of Maryland

Ike C. Ehie
Kansas State University

John K. Fielding
University of Northwestern Ohio

Subodha Kumar
Mays Business School
Texas A&M University

Dan Matthews
Trine University

Avarind Narasipur
Chennai Business School

Nicholas W. Twigg
Coastal Carolina University

Julie Ann Stuart Williams
University of West Florida

We are also indebted to our Product Director, Joe Sabatino; our Product Manager, Aaron Arnsparger; our Marketing Manager, Heather Mooney; our Sr. Content Developer, Maggie Kubale; our Media Developer, Chris Valentine; our Content Project Manager, Jana Lewis, and others at Cengage Learning for their counsel and support during the preparation of this text.

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Dennis J. Sweeney
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About the Authors

David R. Anderson. David R. Anderson is Professor of Quantitative Analysis in the College of Business Administration at the University of Cincinnati. Born in Grand Forks, North Dakota, he earned his B.S., M.S., and Ph.D. degrees from Purdue University. Professor Anderson has served as Head of the Department of Quantitative Analysis and Operations Management and as Associate Dean of the College of Business Administration. In addition, he was the coordinator of the College's first Executive Program.

At the University of Cincinnati, Professor Anderson has taught introductory statistics for business students as well as graduate-level courses in regression analysis, multivariate analysis, and management science. He has also taught statistical courses at the Department of Labor in Washington, D.C. He has been honored with nominations and awards for excellence in teaching and excellence in service to student organizations.

Professor Anderson has coauthored 10 textbooks in the areas of statistics, management science, linear programming, and production and operations management. He is an active consultant in the field of sampling and statistical methods.

Dennis J. Sweeney. Dennis J. Sweeney is Professor of Quantitative Analysis and Founder of the Center for Productivity Improvement at the University of Cincinnati. Born in Des Moines, Iowa, he earned a B.S.B.A. degree from Drake University and his M.B.A. and D.B.A. degrees from Indiana University, where he was an NDEA Fellow. During 1978–79, Professor Sweeney worked in the management science group at Procter & Gamble; during 1981–82, he was a visiting professor at Duke University. Professor Sweeney served as Head of the Department of Quantitative Analysis and as Associate Dean of the College of Business Administration at the University of Cincinnati.

Professor Sweeney has published more than 30 articles and monographs in the area of management science and statistics. The National Science Foundation, IBM, Procter & Gamble, Federated Department Stores, Kroger, and Cincinnati Gas & Electric have funded his research, which has been published in *Management Science, Operations Research, Mathematical Programming, Decision Sciences,* and other journals.

Professor Sweeney has coauthored 10 textbooks in the areas of statistics, management science, linear programming, and production and operations management.

Thomas A. Williams. Thomas A. Williams is Professor of Management Science in the College of Business at Rochester Institute of Technology. Born in Elmira, New York, he earned his B.S. degree at Clarkson University. He did his graduate work at Rensselaer Polytechnic Institute, where he received his M.S. and Ph.D. degrees.

Before joining the College of Business at RIT, Professor Williams served for seven years as a faculty member in the College of Business Administration at the University of Cincinnati, where he developed the undergraduate program in Information Systems and then served as its coordinator. At RIT he was the first chairman of the Decision Sciences Department. He teaches courses in management science and statistics, as well as graduate courses in regression and decision analysis.

Professor Williams is the coauthor of 11 textbooks in the areas of management science, statistics, production and operations management, and mathematics. He has been a consultant for numerous *Fortune* 500 companies and has worked on projects ranging from the use of data analysis to the development of large-scale regression models.

Jeffrey D. Camm Jeffrey D. Camm is Professor of Quantitative Analysis and College of Business Research Fellow in the Carl H. Lindner College of Business at the University of Cincinnati. Born in Cincinnati, Ohio, he holds a B.S. from Xavier University and a Ph.D. from Clemson University. He has been at the University of Cincinnati since 1984, and has been a visiting scholar at Stanford University and a visiting professor of business administration at the Tuck School of Business at Dartmouth College.

Dr. Camm has published over 30 papers in the general area of optimization applied to problems in operations management. He has published his research in *Science*, *Management Science*, *Operations Research*, *Interfaces* and other professional journals. At the University of Cincinnati, he was named the Dornoff Fellow of Teaching Excellence and he was the 2006 recipient of the INFORMS Prize for the Teaching of Operations Research Practice. A firm believer in practicing what he preaches, he has served as an operations research consultant to numerous companies and government agencies. From 2005 to 2010 he served as editor-in-chief of *Interfaces*, and is currently on the editorial board of *INFORMS Transactions on Education*.

James J. Cochran James J. Cochran is Professor of Applied Statistics and the Rogers-Spivey Faculty Fellow in the Department of Information Systems, Statistics, and Management Science at The University of Alabama. Born in Dayton, Ohio, he holds a B.S., an M.S., and an M.B.A. from Wright State University and a Ph.D. from the University of Cincinnati. He has been a visiting scholar at Stanford University, Universidad de Talca, the University of South Africa, and Pole Universitaire Leonard de Vinci.

Professor Cochran has published over 30 papers in the development and application of operations research and statistical methods. He has published his research in *Management Science*, *The American Statistician*, *Communications in Statistics - Theory and Methods*, *European Journal of Operational Research*, *Journal of Combinatorial Optimization*, and other professional journals. He was the 2008 recipient of the INFORMS Prize for the Teaching of Operations Research Practice and the 2010 recipient of the Mu Sigma Rho Statistical Education Award. Professor Cochran was elected to the International Statistics Institute in 2005, named a Fellow of the American Statistical Association in 2011, and received the Founders Award from the American Statistical Association in 2014. A strong advocate for effective operations research and statistics education as a means of improving the quality of applications to real problems, Professor Cochran has organized and chaired teaching effectiveness workshops in Montevideo, Uruguay; Cape Town, South Africa; Cartagena, Colombia; Jaipur, India; Buenos Aires, Argentina; Nairobi, Kenya; Buea, Cameroon; and Osijek, Croatia. He has served as an operations research or statistical consultant to numerous companies and not-for-profit organizations. From 2007 to 2012 Professor Cochran served as editor-in-chief of *INFORMS Transactions on Education*, and he is on the editorial board of several journals including *Interfaces*, the *Journal of the Chilean Institute of Operations Research*, and *ORiON*.

Michael J. Fry Michael J. Fry is Professor and Lindner Research Fellow in the Department of Operations, Business Analytics, and Information Systems in the Carl H. Lindner College of Business at the University of Cincinnati. Born in Killeen, Texas, he earned a B.S. from Texas A&M University, and M.S.E. and Ph.D. degrees from the University of Michigan. He has been at the University of Cincinnati since 2002, and he has been a visiting professor at the Samuel Curtis Johnson Graduate School of Management at Cornell University and the Sauder School of Business at the University of British Columbia.

Professor Fry has published over 20 research publications in journals such as *Operations Research, M&SOM, Transportation Science, Naval Research Logistics, IIE Transactions* and *Interfaces*. His research interests are in applying management science methods to the areas of supply chain analytics, sports analytics, and public-policy operations. He has worked with many different organizations for his research, including Dell, Inc., Copeland Corporation, Starbucks Coffee Company, the Cincinnati Fire Department, the State of Ohio Election Commission, the Cincinnati Bengals, and the Cincinnati Zoo. Professor Fry has won multiple teaching awards including the 2013 Michael L. Dean Excellence in Graduate Teaching Award and the 2006 Daniel J. Westerbeck Junior Faculty Teaching Award.

Jeffrey W. Ohlmann Jeffrey W. Ohlmann is Associate Professor of Management Sciences in the Tippie College of Business at the University of Iowa. Born in Valentine, Nebraska, he earned a B.S. from the University of Nebraska, and M.S. and Ph.D. degrees from the University of Michigan. He has been at the University of Iowa since 2003.

Professor Ohlmann's research on the modeling and solution of decision-making problems has produced over a dozen research papers in journals such as *Mathematics of Operations Research*, *INFORMS Journal on Computing*, *Transportation Science*, and *Interfaces*. He has collaborated with companies such as Transfreight, LeanCor, Cargill, the Hamilton County Board of Elections, and the Cincinnati Bengals. Due to the relevance of his work to industry, he was bestowed the George B. Dantzig Dissertation Award and was recognized as a finalist for the Daniel H. Wagner Prize for Excellence in Operations Research Practice.

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CHAPTER 1

Introduction

CONTENTS

Management science, an approach to decision making based on the scientific method, makes extensive use of quantitative analysis. A variety of names exists for the body of knowledge involving quantitative approaches to decision making; in addition to management science, two other widely known and accepted names are operations research and decision science. Today, many use the terms *management science, operations research,* and *decision science* interchangeably.

The scientific management revolution of the early 1900s, initiated by Frederic W. Taylor, provided the foundation for the use of quantitative methods in management. But modern management science research is generally considered to have originated during the World War II period, when teams were formed to deal with strategic and tactical problems faced by the military. These teams, which often consisted of people with diverse specialties (e.g., mathematicians, engineers, and behavioral scientists), were joined together to solve a common problem by utilizing the scientific method. After the war, many of these team members continued their research in the field of management science.

Two developments that occurred during the post–World War II period led to the growth and use of management science in nonmilitary applications. First, continued research resulted in numerous methodological developments. Probably the most significant development was the discovery by George Dantzig, in 1947, of the simplex method for solving linear programming problems. At the same time these methodological developments were taking place, digital computers prompted a virtual explosion in computing power. Computers enabled practitioners to use the methodological advances to solve a large variety of problems. The computer technology explosion continues; smart phones, tablets and other mobile-computing devices can now be used to solve problems larger than those solved on mainframe computers in the 1990s.

More recently, the explosive growth of data from sources such as smart phones and other personal-electronic devices provide access to much more data today than ever before. Additionally, the internet allows for easy sharing and storage of data, providing extensive access to a variety of users to the necessary inputs to management-science models.

As stated in the Preface, the purpose of the text is to provide students with a sound conceptual understanding of the role that management science plays in the decision-making process. We also said that the text is applications oriented. To reinforce the applications nature of the text and provide a better understanding of the variety of applications in which management science has been used successfully, Management Science in Action articles are presented throughout the text. Each Management Science in Action article summarizes an application of management science in practice. The first Management Science in Action in this chapter, Revenue Management at AT&T Park, describes one of the most important applications of management science in the sports and entertainment industry.

MANAGEMENT SCIENCE IN ACTION

REVENUE MANAGEMENT AT AT&T PARK*

Imagine the difficult position Russ Stanley, Vice President of Ticket Services for the San Francisco Giants, found himself facing late in the 2010 baseball season. Prior to the season, his organization had adopted a dynamic approach to pricing its tickets similar to the model successfully pioneered by Thomas M. Cook and his operations research group at American Airlines. Stanley desparately wanted the Giants to clinch a playoff birth, but he didn't want the team to do so *too quickly*.

When dynamically pricing a good or service, an organization regularly reviews supply and demand of the product and uses operations research to determine if the price should be changed to reflect these conditions. As the scheduled takeoff date for a flight nears, the cost of a ticket increases if seats for the flight are relatively scarce. On the other hand, the airline discounts tickets for an approaching flight with relatively few ticketed passengers. Through the use of optimization to dynamically set ticket prices, American Airlines generates nearly $1 billion annually in incremental revenue.

The management team of the San Francisco Giants recognized similarities between their primary product (tickets to home games) and the primary product sold by airlines (tickets for flights) and adopted a similar revenue management system. If a particular Giants' game is appealing to fans, tickets sell quickly and demand far exceeds supply as the date of the game approaches; under these conditions fans will be willing to pay more and the Giants charge a premium for the ticket. Similarly, tickets for less attractive games are discounted to reflect relatively low demand by fans. This is why Stanley found himself in a quandary at the end of the 2010 baseball season. The Giants were in the middle of a tight pennant race with the San Diego Padres that effectively increased demand for tickets to Giants' games, and the team was actually scheduled to play the Padres in San Francisco for the last three games of the season. While Stanley certainly wanted his club to win its division and reach the Major League Baseball playoffs, he also recognized that his team's revenues would be greatly enhanced if it didn't qualify for the playoffs until the last day of the season. "I guess financially it is better to go all the way down to the last game," Stanley said in a late season interview. "Our hearts are in our stomachs; we're pacing watching these games."

Does revenue management and operations research work? Today, virtually every airline uses some sort of revenue-management system, and the cruise, hotel, and car rental industries also now apply revenue-management methods. As for the Giants, Stanley said dynamic pricing provided a 7% to 8% increase in revenue per seat for Giants' home games during the 2010 season. Coincidentally, the Giants did win the National League West division on the last day of the season and ultimately won the World Series. Several professional sports franchises are now looking to the Giants' example and considering implementation of similar dynamic ticket-pricing systems.

*Based on Peter Horner, "The Sabre Story," *OR/MS Today* (June 2000); Ken Belson, "Baseball Tickets Too Much? Check Back Tomorrow," *NewYork Times.com* (May 18, 2009); and Rob Gloster, "Giants Quadruple Price of Cheap Seats as Playoffs Drive Demand," *Bloomberg Business-week* (September 30, 2010).

1.1 PROBLEM SOLVING AND DECISION MAKING

Problem solving can be defined as the process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve the difference. For problems important enough to justify the time and effort of careful analysis, the problem-solving process involves the following seven steps:

1. Identify and define the problem.
2. Determine the set of alternative solutions.
3. Determine the criterion or criteria that will be used to evaluate the alternatives.
4. Evaluate the alternatives.
5. Choose an alternative.
6. Implement the selected alternative.
7. Evaluate the results to determine whether a satisfactory solution has been obtained.

Decision making is the term generally associated with the first five steps of the problem-solving process. Thus, the first step of decision making is to identify and define the problem. Decision making ends with the choosing of an alternative, which is the act of making the decision.

Let us consider the following example of the decision-making process. For the moment assume that you are currently unemployed and that you would like a position that will lead to a satisfying career. Suppose that your job search has resulted in offers from companies in Rochester, New York; Dallas, Texas; Greensboro, North Carolina; and Pittsburgh, Pennsylvania. Thus, the alternatives for your decision problem can be stated as follows:

1. Accept the position in Rochester.
2. Accept the position in Dallas.

3. Accept the position in Greensboro.
4. Accept the position in Pittsburgh.

The next step of the problem-solving process involves determining the criteria that will be used to evaluate the four alternatives. Obviously, the starting salary is a factor of some importance. If salary were the only criterion of importance to you, the alternative selected as "best" would be the one with the highest starting salary. Problems in which the objective is to find the best solution with respect to one criterion are referred to as **single-criterion decision problems.**

Suppose that you also conclude that the potential for advancement and the location of the job are two other criteria of major importance. Thus, the three criteria in your decision problem are starting salary, potential for advancement, and location. Problems that involve more than one criterion are referred to as **multicriteria decision problems.**

The next step of the decision-making process is to evaluate each of the alternatives with respect to each criterion. For example, evaluating each alternative relative to the starting salary criterion is done simply by recording the starting salary for each job alternative. Evaluating each alternative with respect to the potential for advancement and the location of the job is more difficult to do, however, because these evaluations are based primarily on subjective factors that are often difficult to quantify. Suppose for now that you decide to measure potential for advancement and job location by rating each of these criteria as poor, fair, average, good, or excellent. The data that you compile are shown in Table 1.1.

You are now ready to make a choice from the available alternatives. What makes this choice phase so difficult is that the criteria are probably not all equally important, and no one alternative is "best" with regard to all criteria. Although we will present a method for dealing with situations like this one later in the text, for now let us suppose that after a careful evaluation of the data in Table 1.1, you decide to select alternative 3; alternative 3 is thus referred to as the **decision.**

At this point in time, the decision-making process is complete. In summary, we see that this process involves five steps:

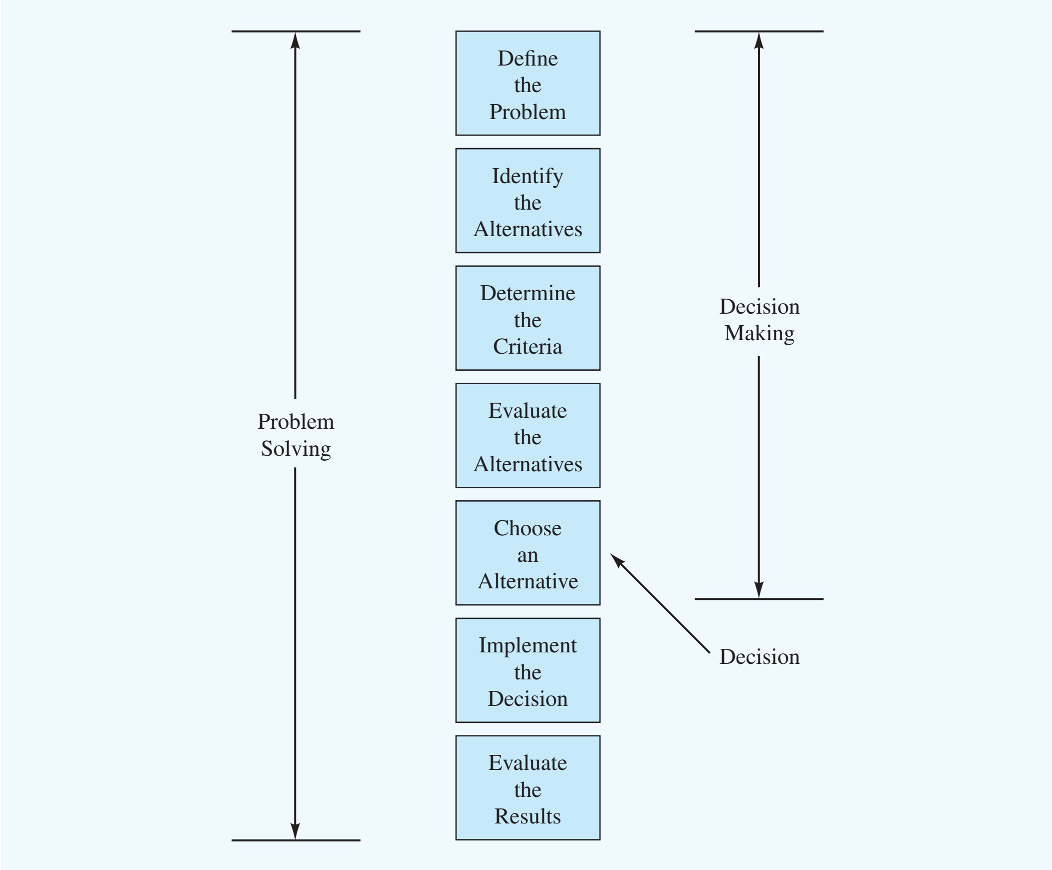
1. Define the problem.
2. Identify the alternatives.
3. Determine the criteria.
4. Evaluate the alternatives.
5. Choose an alternative.

Note that missing from this list are the last two steps in the problem-solving process: implementing the selected alternative and evaluating the results to determine whether a satisfactory solution has been obtained. This omission is not meant to diminish the importance of each of these activities, but to emphasize the more limited scope of the term *decision making* as compared to the term *problem solving*. Figure 1.1 summarizes the relationship between these two concepts.

TABLE 1.1 DATA FOR THE JOB EVALUATION DECISION-MAKING PROBLEM

Alternative	Starting Salary	Potential for Advancement	Job Location
1. Rochester	$48,500	Average	Average
2. Dallas	$46,000	Excellent	Good
3. Greensboro	$46,000	Good	Excellent
4. Pittsburgh	$47,000	Average	Good

FIGURE 1.1 THE RELATIONSHIP BETWEEN PROBLEM SOLVING AND DECISION MAKING



1.2 QUANTITATIVE ANALYSIS AND DECISION MAKING

Consider the flowchart presented in Figure 1.2. Note that it combines the first three steps of the decision-making process under the heading of "Structuring the Problem" and the latter two steps under the heading "Analyzing the Problem." Let us now consider in greater detail how to carry out the set of activities that make up the decision-making process.

FIGURE 1.2 AN ALTERNATE CLASSIFICATION OF THE DECISION-MAKING PROCESS

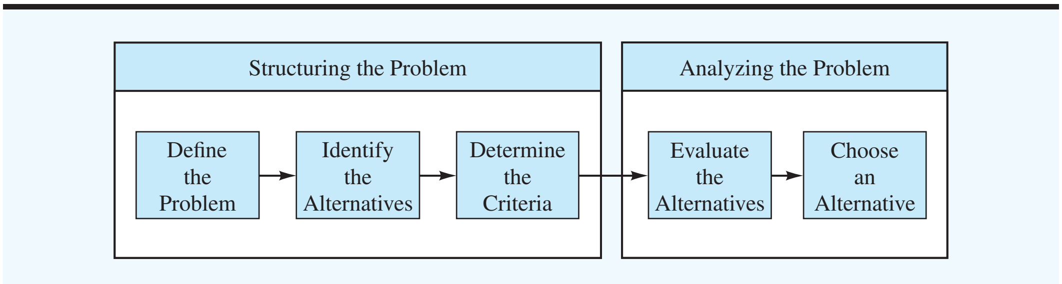


Figure 1.3 shows that the analysis phase of the decision-making process may take two basic forms: qualitative and quantitative. Qualitative analysis is based primarily on the manager's judgment and experience; it includes the manager's intuitive "feel" for the problem and is more an art than a science. If the manager has had experience with similar problems or if the problem is relatively simple, heavy emphasis may be placed upon a qualitative analysis. However, if the manager has had little experience with similar problems, or if the problem is sufficiently complex, then a quantitative analysis of the problem can be an especially important consideration in the manager's final decision.

When using the quantitative approach, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions that describe the objectives, constraints, and other relationships that exist in the problem. Then, by using one or more quantitative methods, the analyst will make a recommendation based on the quantitative aspects of the problem.

Although skills in the qualitative approach are inherent in the manager and usually increase with experience, the skills of the quantitative approach can be learned only by studying the assumptions and methods of management science. A manager can increase decision-making effectiveness by learning more about quantitative methodology and by better understanding its contribution to the decision-making process. A manager who is knowledgeable in quantitative decision-making procedures is in a much better position to compare and evaluate the qualitative and quantitative sources of recommendations and ultimately to combine the two sources in order to make the best possible decision.

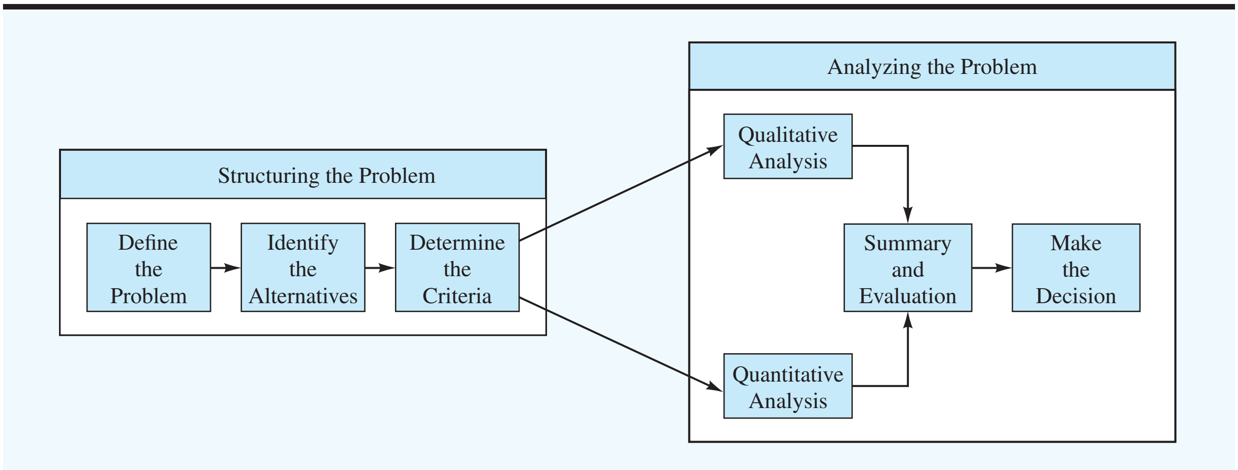
The box in Figure 1.3 entitled "Quantitative Analysis" encompasses most of the subject matter of this text. We will consider a managerial problem, introduce the appropriate quantitative methodology, and then develop the recommended decision.

In closing this section, let us briefly state some of the reasons why a quantitative approach might be used in the decision-making process:

Try Problem 4 to test your understanding of why quantitative approaches might be needed in a particular problem.

1. The problem is complex, and the manager cannot develop a good solution without the aid of quantitative analysis.
2. The problem is especially important (e.g., a great deal of money is involved), and the manager desires a thorough analysis before attempting to make a decision.

FIGURE 1.3 THE ROLE OF QUALITATIVE AND QUANTITATIVE ANALYSIS



3. The problem is new, and the manager has no previous experience from which to draw.
4. The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to make routine decision recommendations.

1.3 QUANTITATIVE ANALYSIS

From Figure 1.3, we see that quantitative analysis begins once the problem has been structured. It usually takes imagination, teamwork, and considerable effort to transform a rather general problem description into a well-defined problem that can be approached via quantitative analysis. The more the analyst is involved in the process of structuring the problem, the more likely the ensuing quantitative analysis will make an important contribution to the decision-making process.

To successfully apply quantitative analysis to decision making, the management scientist must work closely with the manager or user of the results. When both the management scientist and the manager agree that the problem has been adequately structured, work can begin on developing a model to represent the problem mathematically. Solution procedures can then be employed to find the best solution for the model. This best solution for the model then becomes a recommendation to the decision maker. The process of developing and solving models is the essence of the quantitative analysis process.

Model Development

Models are representations of real objects or situations and can be presented in various forms. For example, a scale model of an airplane is a representation of a real airplane. Similarly, a child's toy truck is a model of a real truck. The model airplane and toy truck are examples of models that are physical replicas of real objects. In modeling terminology, physical replicas are referred to as **iconic models.**

A second classification includes models that are physical in form but do not have the same physical appearance as the object being modeled. Such models are referred to as **analog models.** The speedometer of an automobile is an analog model; the position of the needle on the dial represents the speed of the automobile. A thermometer is another analog model representing temperature.

A third classification of models—the type we will primarily be studying—includes representations of a problem by a system of symbols and mathematical relationships or expressions. Such models are referred to as **mathematical models** and are a critical part of any quantitative approach to decision making. For example, the total profit from the sale of a product can be determined by multiplying the profit per unit by the quantity sold. If we let x represent the number of units sold and P the total profit, then, with a profit of \$10 per unit, the following mathematical model defines the total profit earned by selling x units:

$$P = 10x \tag{1.1}$$

The purpose, or value, of any model is that it enables us to make inferences about the real situation by studying and analyzing the model. For example, an airplane designer might test an iconic model of a new airplane in a wind tunnel to learn about the potential flying characteristics of the full-size airplane. Similarly, a mathematical model may be used to make inferences about how much profit will be earned if a specified quantity of a particular product is sold. According to the mathematical model of equation (1.1), we would expect selling three units of the product ($x = 3$) would provide a profit of $P = 10(3) = \$30$.

In general, experimenting with models requires less time and is less expensive than experimenting with the real object or situation. A model airplane is certainly quicker and less expensive to build and study than the full-size airplane. Similarly, the mathematical model in equation (1.1) allows a quick identification of profit expectations without actually requiring the manager to produce and sell x units. Models also have the advantage of reducing the risk associated with experimenting with the real situation. In particular, bad designs or bad decisions that cause the model airplane to crash or a mathematical model to project a \$10,000 loss can be avoided in the real situation.

Herbert A. Simon, a Nobel Prize winner in economics and an expert in decision making, said that a mathematical model does not have to be exact; it just has to be close enough to provide better results than can be obtained by common sense.

The value of model-based conclusions and decisions is dependent on how well the model represents the real situation. The more closely the model airplane represents the real airplane, the more accurate the conclusions and predictions will be. Similarly, the more closely the mathematical model represents the company's true profit-volume relationship, the more accurate the profit projections will be.

Because this text deals with quantitative analysis based on mathematical models, let us look more closely at the mathematical modeling process. When initially considering a managerial problem, we usually find that the problem definition phase leads to a specific objective, such as maximization of profit or minimization of cost, and possibly a set of restrictions or **constraints,** such as production capacities. The success of the mathematical model and quantitative approach will depend heavily on how accurately the objective and constraints can be expressed in terms of mathematical equations or relationships.

A mathematical expression that describes the problem's objective is referred to as the **objective function.** For example, the profit equation $P = 10x$ would be an objective function for a firm attempting to maximize profit. A production capacity constraint would be necessary if, for instance, 5 hours are required to produce each unit and only 40 hours of production time are available per week. Let x indicate the number of units produced each week. The production time constraint is given by

$$5x \leq 40 \tag{1.2}$$

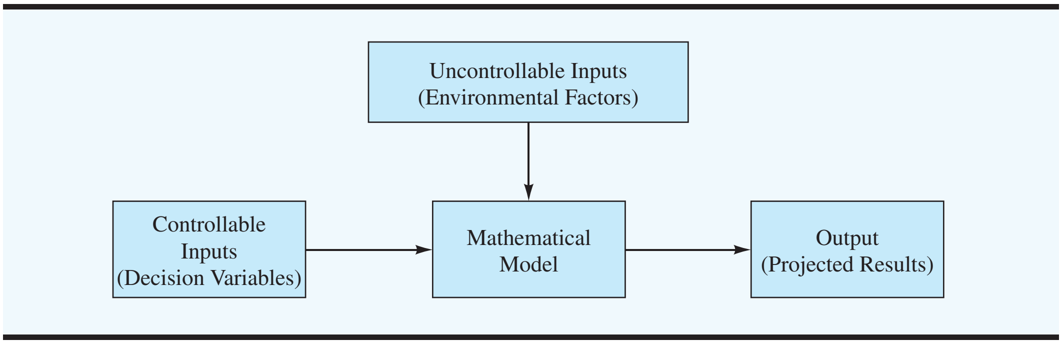
The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.

The decision problem or question is the following: How many units of the product should be scheduled each week to maximize profit? A complete mathematical model for this simple production problem is

$$\begin{aligned} &\text{Maximize} \quad P = 10x \quad \text{objective function} \\ &\text{subject to (s.t.)} \\ &\qquad\qquad \left.\begin{aligned} 5x &\leq 40 \\ x &\geq 0 \end{aligned}\right\} \text{constraints} \end{aligned}$$

The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number of units. The optimal solution to this model can be easily calculated and is given by $x = 8$, with an associated profit of \$80. This model is an example of a linear programming

FIGURE 1.4 FLOWCHART OF THE PROCESS OF TRANSFORMING MODEL INPUTS INTO OUTPUT



model. In subsequent chapters we will discuss more complicated mathematical models and learn how to solve them in situations where the answers are not nearly so obvious.

In the preceding mathematical model, the profit per unit ($10), the production time per unit (5 hours), and the production capacity (40 hours) are environmental factors that are not under the control of the manager or decision maker. Such environmental factors, which can affect both the objective function and the constraints, are referred to as **uncontrollable inputs** to the model. Inputs that are completely controlled or determined by the decision maker are referred to as **controllable inputs** to the model. In the example given, the production quantity x is the controllable input to the model. Controllable inputs are the decision alternatives specified by the manager and thus are also referred to as the **decision variables** of the model.

Once all controllable and uncontrollable inputs are specified, the objective function and constraints can be evaluated and the output of the model determined. In this sense, the output of the model is simply the projection of what would happen if those particular environmental factors and decisions occurred in the real situation. A flowchart of how controllable and uncontrollable inputs are transformed by the mathematical model into output is shown in Figure 1.4. A similar flowchart showing the specific details of the production model is shown in Figure 1.5.

FIGURE 1.5 FLOWCHART FOR THE PRODUCTION MODEL